

The Coordinate Plane Resolution and Adjustment for Roots Determination

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Abstract:

This study is a geometric root determination method for quadratic equations which contain a theoretical proof of the quadratic formula and gives an illustrative evidence of Euclid's fifth postulate.

The core notion is that a quadratic function whose roots are to be found given as $F(x)$, have a correspondent perfect square whose first two terms when conventionally made equal to that of $F(x)$ will form a resultant function with a geometric property that each term as a component function, from the least to the greatest power of x , can reconstruct a frame for the formation of the next component function until the zeroes are achieved, then the function $F(x)$ is manipulated likewise with each term on this geometric representation formed by the correspondent perfect square to give the roots.

This paper offers the privilege to work separately with the terms of a quadratic equation to locate the roots.

Keywords: Perfect Square, Component Functions, Roots, Quadratic Formula, Parallel Postulate.

1. INTRODUCTION

BACKGROUND OF THE STUDY

This study is based on the derivation of a new quadratic root determination method and the illustration of Euclid's fifth postulate, with the latter many mathematicians think it is not self-evident although it is indisputable. A proof should be rendered when a postulate is not all that self-evident. Proofs rendered were discarded with time. Proclus in the fifth century stated that, "The postulate makes a positive statement about a region beyond the reach of possible observation or geometrical intuition". With this being an advice to think out of the box in this case, other than dwelling solely on the foundations laid down.

Hence the parallel postulate cannot be proved based on the current understanding of geometry or basically the first four postulates of Euclid. The first four postulates nearly have some direct geometrical implementation and deduction on the Cartesian plane, just the Cartesian plane with basically no other drawings, but the fifth postulate does not have any explicit geometric deductions from the Cartesian plane, and again just the planes.

Mathematicians involve certain identities, in certain problems by the introduction of some proof pertaining to that identity, because the fifth postulate cannot be traced explicitly from the Cartesian plane and directly from the first four postulates then there should be this proof to enable us to consider its involvement. To achieve this there should be a deeper thought outside the box to achieve this beauty. Hence this leads us to The Coordinate Plane Resolution and Adjustment for Roots Determination. Surprisingly this study was not triggered by the taste to find the proof of the fifth postulate but rather the hope in finding an alternative proof for the quadratic formula which is solely geometric in nature. Then after the success in alternatively proving the quadratic formula, the fifth postulate was geometrically achieved surprisingly showing that the parallel postulate is a key hidden property of geometric shapes and this study unlocks it.

It is known that completing by square is a popular proof of quadratic equations, although not faulty, this study introduces a way of deriving the quadratic formula. The additions as far as knowledge is concerned are that this study dwells on the separate geometry of the terms of a quadratic equation to determine the roots of quadratic equations.

THE PROBLEM OF THE STUDY

- The problem at hand is that there is a limitation to the modern day geometric analysis, as their blueprint is merely thought to be the perpendicular lines from the X and Y axis drawn to trace or locate the points satisfying a particular equation.
- One problem is also about bringing forth an alternative proof to the quadratic formula and an alternative roots determination method.
- The proof of the fifth postulate geometrically is surely a key problem.

OBJECTIVES

- To prove the existence of the fifth postulate of Euclid directly from the Cartesian plane.
- To find an alternative roots determination method for quadratic equations.
- To be able to understand curves (quadratic curves).

RESEARCH QUESTIONS

- How reliable is the study?
- Is there any alternative proof to the quadratic formula?
- Are geometric curves detailed?

THE PARALLEL POSTULATE

Surprisingly the coordinate plane resolution and adjustment for roots determination uses the fifth postulate, something Euclid (300BC) never did as he used only the first four postulates. **Fig 2.3** of the procedure and the proof of the quadratic formula gives evidence of basically the key significance of the di-axis relationship as explained in the parallel postulate. The parallel postulate states that, “When two lines are produced on a third line such that the sum of interior angles (α and β) they make with the third line is less than two right-angles, then when these two lines are extended further on the side on which they form their angles on the third line, then they will eventually meet”.

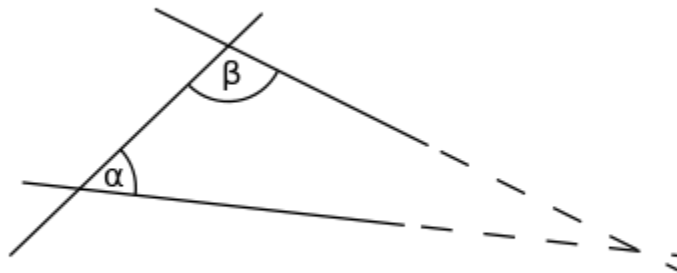


Fig.1.1; The parallel postulate.

DEFINITION 1.1 The Component Functions: In a given quadratic equation $F(x)$, the terms of $F(x)$ are made the component functions when each term is given a separate geometric representation on the Cartesian plane. Such that for any given quadratic function

$$F(x) = a_2x^2 + a_1x^1 + a_0x^0 \dots\dots\dots (1)$$

The notation $f_2(x) = a_2x^2$, $f_1(x) = a_1x^1$ and $f_0(x) = a_0x^0$ are the component functions whose variable given by x has power two, one and zero respectively. Bear in mind that a_0 , a_1 and a_2 are coefficients of x^0 , x^1 and x^2 respectively.

DEFINITION 1.2 Correspondent Perfect Square of a Quadratic Equation: This is a perfect square whose roots are basically the average of the roots of an infinite number of quadratic equations all having equal component functions; $f_1(x) = a_1x^1$ and $f_2(x) = a_2x^2$, but with difference in their component function $f_0(x) = a_0x^0$, surprisingly the correspondent perfect square is a part of this infinite members so its $f_1(x) = a_1x^1$ and $f_2(x) = a_2x^2$ are equivalent to that of the rest of functions with similar property, but differs in $f_0(x) = a_0x^0$. In detail, when a function is considered the correspondent perfect square then it has $f_0(x) = a_0x^0$ to be $f_0(x) = \frac{(a_1)^2}{4a_2}$. As it is the square of the first derivative of a given quadratic function $F(x)$ and therefore can be surely found from at least one function in the group.

1.3 The Coordinate Plane Resolutions and Adjustments for Root Determination: This title is the general idea in this text, as the name goes can be seen to be a distinct study for roots determination. Clearly the name depicts that it is a geometric method for roots determination.

DEFINITION 1.3 The Coordinate Plane Resolutions and Adjustments for Root Determination: This is a geometric process in roots determination whereby each term is a component function of some quadratic function given by $F(x)$ from equation (1) above, and that $F(x)$ has a correspondent perfect square given by;

$$\frac{F^1(x)^2}{4a_2} = a_2x^2 + a_1x^1 + \frac{(a_1)^2}{4a_2} \dots\dots\dots (2)$$

The terms of $\frac{F^1(x)^2}{4a_2}$ can sequentially be taken from the least to the greatest power of X , so that a frame can be resolved from the arbitrary XY plane using $f_0(x) - y = \frac{(a_1)^2}{4a_2} - y$ for the formation of the next component functions which are $f_1(x) = a_1x^1$ and then $f_2(x) = a_2x^2$, until the zeroes of $\frac{F^1(x)^2}{4a_2}$ are achieved, from there the quadratic function initially given as $F(x)$ would have its respective terms separately manipulated on the resultant geometric representation formed by $\frac{F^1(x)^2}{4a_2}$, following the procedure in section 2.1, to give the roots.

2. METHODOLOGY:

This method uses the property of the Cartesian plane to basically generate a way to find the roots of a quadratic equation. In this section, first there would be elaborations on the backbone of the study given as the conditions then the procedure and the proof of the quadratic formula would be dealt with, along with a couple of examples. A quadratic equation can be given in the form $F(x) = a_2x^2 + a_1x^1 + a_0x^0$

Conditions to Be Satisfied when using the component functions.

Condition 1 working with the component function $f_0(x) = a_0x^0$:

Given that $F(x) = a_2x^2 + a_1x^1 + a_0x^0$, where $-\infty \leq a_0x^0 \leq \infty$, these functions constitute a group of quadratic equations with the same average roots when the terms a_2x^2 and a_1x^1 are the same hence they have the same correspondent perfect square given as; $\frac{F^1(x)^2}{4a_2} = a_2x^2 + a_1x^1 + \frac{(a_1)^2}{4a_2}$. So for the roots determination to be possible, as the zeroes of the correspondent perfect square are the initial roots before any manipulation with $F(x)$ to locate their respective roots, hence the correspondent perfect square would first translate the XY plane, the translation is given by $y^1 = \frac{(a_1)^2}{4a_2} - y$.

Proof:

Therefore given $F(x) = a_2x^2 + a_1x^1 + a_0x^0$ for $F(x) = y$.

As part of the quest in finding the roots of a particular function $F(x)$, first of all the zeroes of the correspondent perfect square $\frac{F^1(x)^2}{4a_2}$ is supposed to be found.

To get $\frac{F^1(x)^2}{4a_2}$, $F(x)$ is first differentiated giving;

$$F^1(x) = 2a_2x + a_1 \dots\dots\dots (3)$$

Then both side of equation (3) is squared thus giving;

$$F^1(x)^2 = 4(a_2)^2x^2 + 4a_2a_1x + (a_1)^2$$

The terms of the squared derivative is supposed to have its non-constant terms made equal to that of $F(x)$ so both side of the equation is divided by $4a_2$ hence resulting in the correspondent perfect square.

This correspondent perfect square is finally given by;

$$\frac{F^1(x)^2}{4a_2} = a_2x^2 + a_1x^1 + \frac{(a_1)^2}{4a_2}$$

Note that this same method used in getting the correspondent perfect square is applied in the procedure and the proof of the quadratic formula in section 2.1, step 1.

As known that to find the roots of a particular equation, it is very obligatory to make $F(x) = 0$ for the quadratic function in question but then do not equate $\frac{F^1(x)^2}{4a_2}$ to zero for the correspondent perfect square too. Because the curve of $F(x)$ from which its roots are determine has the zeroes of the correspondent perfect square assigned some y values in correlation.

As the correspondent perfect square $\frac{F^1(x)^2}{4a_2}$ can be considered a function $G(x)$, so make $G(x) = y$

$$\text{Therefore; } y = a_2x^2 + a_1x^1 + \frac{(a_1)^2}{4a_2}$$

$$\text{Then; } 0 = a_2x^2 + a_1x^1 + \frac{(a_1)^2}{4a_2} - y$$

Hence the component function whose variable has power zero is observed to be modified as;

$$a_0x^0 - y = \frac{(a_1)^2}{4a_2} - y$$

Therefore a translation is guaranteed in the y-axis given by; $y^1 = \frac{(a_1)^2}{4a_2} - y$

Condition 2 working with the component function $f_2(x) = a_2x^2$:

The translation of the curve $y^1 = a_2x^2$ which is the component function $f_2(x) = a_2x^2$ of the main function given

by $F(x) = a_2x^2 + a_1x^1 + a_0x^0$ as shown in fig 2.1 into $y^1 = a_2 \left(x + \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0x^0}{a_2}} \right)^2$ and

$y^1 = a_2 \left(x - \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0x^0}{a_2}} \right)^2$ is symbolic to the correlation of the difference in the line $y = \frac{(a_1)^2}{4a_2}$ of the

correspondent perfect square and line $y = a_0x^0$ of the quadratic equation whose roots are to be determined with

the X axis on the curve $y^1 = a_2x^2$, hence the difference is given as line $y^1 = \frac{(a_1)^2}{4a_2} - a_0x^0$ on the curve $y^1 = a_2x^2$

and in addition $y^1 = \frac{(a_1)^2}{4a_2} - a_0x^0$ has some corresponding X values given as $x = \pm \sqrt{\left(\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0x^0}{a_2} \right)}$ on this

same curve $y^1 = a_2x^2$.

Hence basically there is a translation of the vector $\begin{pmatrix} x \\ y^1 \end{pmatrix}$ given by $\begin{pmatrix} x \\ y^1 \end{pmatrix} \rightarrow \begin{pmatrix} x + \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0x^0}{a_2}} \\ y^1 \end{pmatrix}$ and $\begin{pmatrix} x \\ y^1 \end{pmatrix} \rightarrow$

$\begin{pmatrix} x - \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0x^0}{a_2}} \\ y^1 \end{pmatrix}$ in the curve $y^1 = a_2x^2$.

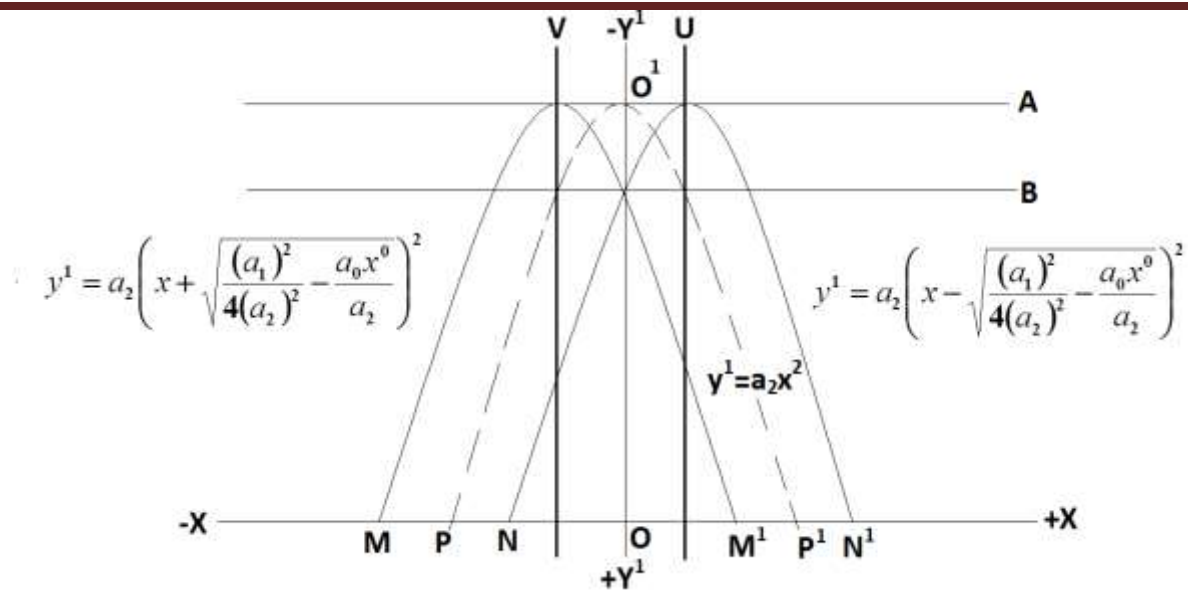


Fig 2.1; component function drawn in reference to the XY^1 plane.

The reference with O and O^1 as origins in the XY plane and XY^1 plane respectively are not considered simultaneously.

Line A is given by $y^1 = 0$ or $y = \frac{(a_1)^2}{4a_2}$ and line B is given by $y^1 = \frac{(a_1)^2}{4a_2} - a_0 x^0$ or line $y = a_0 x^0$

Line U is given by $x = \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}$ and line V is given by $x = -\sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}$ note that basically line

U and line V correlates with line B on the curve $y^1 = a_2 x^2$

Point P is given by $P = \left(-\frac{a_1}{2a_2}, \frac{(a_1)^2}{4a_2}\right)$ and Point P^1 is given by $P^1 = \left(\frac{a_1}{2a_2}, \frac{(a_1)^2}{4a_2}\right)$ in reference to O^1

O the arbitrary chosen origin is now given by $O = \left(0, \frac{(a_1)^2}{4a_2}\right)$ in reference to the XY^1 plane. From the diagram

above line U and line V become new axes for the curve $y^1 = a_2 x^2$. Hence $y^1 = a_2 x^2$ is translated into curves

$$y^1 = a_2 \left(x + \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}\right)^2 \text{ and } y^1 = a_2 \left(x - \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}\right)^2$$

Bear in mind that when taking reference in the XY plane, the reference taken in the XY^1 plane is not taken into consideration and vice versa.

Condition 3 working with the component function $f_1(x) = a_1 x^1$:

From **condition 2** deduce that as $y^1 = a_2 \left(x + \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}\right)^2$ and $y^1 = a_2 \left(x - \sqrt{\frac{(a_1)^2}{4(a_2)^2} - \frac{a_0 x^0}{a_2}}\right)^2$ then their

roots can be sourced from $y = 0$ in the XY plane or $y^1 = \frac{(a_1)^2}{4a_2}$ in the XY^1 plane then the corresponding X

values (the zeroes) become $x = \pm \frac{a_1}{2a_2} + \sqrt{\frac{(a_1)^2}{(4a_2)^2} - \frac{a_0x^0}{a_2}}$ and $x = \pm \frac{a_1}{2a_2} - \sqrt{\frac{(a_1)^2}{(4a_2)^2} - \frac{a_0x^0}{a_2}}$ such that

$D = \pm \sqrt{\frac{(a_1)^2}{(4a_2)^2} - \frac{a_0x^0}{a_2}}$. But then there cannot be four X values of a quadratic equation so in this case there

would be sign alternation of the X values at the points of intersections between the curve $y^1 = a_2x^2$ and the X axis at line $y^1 = \frac{(a_1)^2}{4a_2}$ or $y = 0$, these four possible intersections thus have, $x = -D - \frac{(a_1)}{(2a_2)}$ and $x = D - \frac{(a_1)}{(2a_2)}$

which have positive gradient, and then $x = -D + \frac{(a_1)}{(2a_2)}$ and $x = D + \frac{(a_1)}{(2a_2)}$ which has a negative gradient on the

curve $y^1 = a_2x^2$, Where $x = -\frac{a_1}{2a_2}$ is the root of the correspondent perfect square. With the sign alternation, so

called because the sign on the gradient of the normal of $y^1 = a_1x^1$ pertaining to the component function $f_1(x) = a_1x^1$ given by $x = -a_1y$ in this case negative, would have the signs on the gradient of $x = -D - \frac{(a_1)}{(2a_2)}$,

$x = D - \frac{(a_1)}{(2a_2)}$, $x = -D + \frac{(a_1)}{(2a_2)}$ and $x = D + \frac{(a_1)}{(2a_2)}$ on the curve $y^1 = a_2x^2$ to conform to the negative sign. Hence

the root of the given quadratic equation becomes $x = D - \frac{(a_1)}{(2a_2)}$ and $x = -D - \frac{(a_1)}{(2a_2)}$ the X values with negative

gradient on the curve $y^1 = a_2x^2$. If the gradient of the normal is positive then the X values with negative gradient is negated or the X values with positive gradients are picked as the roots of the quadratic equation.

By combining the conditions the appropriate way of locating the roots of a given quadratic equation after the generalization of the method is effectively delivered, in the next section the general process of doing this is stated in a stepwise numeric order systematically.

2.1 THE PROCEDURE AND THE PROOF OF THE QUADRATIC FORMULA.

The quadratic formula gives a general solution to quadratic equations and has been shown that the technique of completing the square is popularly used in proving the quadratic formula, but if this method does succeed in proving the quadratic formula after the procedure is laid down then, it is reliable in determining the roots of all quadratic functions.

The quadratic formula is proved using the procedure below by sequentially following the steps.

Solving,

$F(x) = Ax^2 + Bx^1 + Cx^0$. Where A , B and C the coefficients of x , and $A = a_2$, $B = a_1$ and $C = a_0$. But for the sake of the standard coefficients in the quadratic formula make a_2 , a_1 and a_0 be represented as a , b and c respectively.

Hence the function becomes $F(x) = ax^2 + bx + c$ (4)

STEP 1: The derivation of the correspondent perfect square from a function given in equation (4) by;

$$F(x) = ax^2 + bx + c$$

$\frac{F^1(x)^2}{4a}$ as obtained from the function $F(x)$ is given by;

$$\frac{F^1(x)^2}{4a} = ax^2 + bx + \frac{b^2}{4a}. \text{ This is the correspondent perfect square of } F(x).$$

Therefore $0 = ax^2 + bx + \frac{b^2}{4a} - y$, will have its component functions operated on to form a geometric representation which would lead to the roots determination of the given function $F(x)$.

STEP 2: Define the component functions of $\frac{F^1(x)^2}{4a}$ as $f_0(x) = \frac{b^2}{4a}$, $f_1(x) = bx$ and $f_2(x) = ax^2$.

STEP 3: Translate the XY plane to XY^1 plane with $f_0(x) - y = \frac{b^2}{4a} - y$ or $y^1 = \frac{b^2}{4a} - y$ from the correspondent perfect square. Fig 2.2 depicts that on the XY plane the location of O^1 is given by point $\left(0, \frac{b^2}{4a}\right)$ while O is the origin and similarly on the XY^1 plane the location of O becomes point $\left(0, \frac{b^2}{4a}\right)$ while O^1 is the origin.

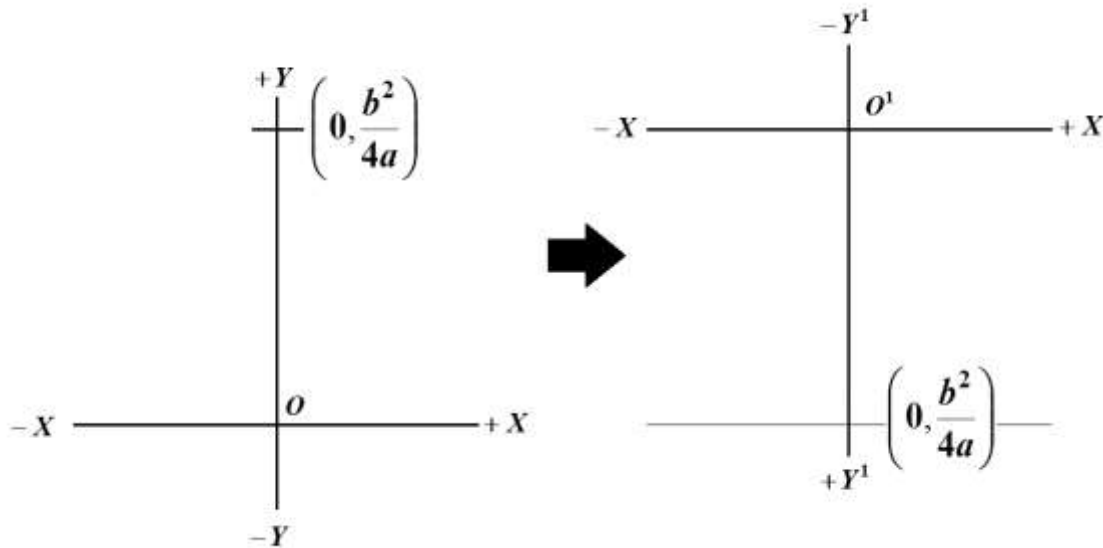


Fig.2.2; The resolution of the XY^1 plane

STEP 4: Draw line $y^1 = bx$ from the component function $f_1(x) = bx$, and its perpendicular line $x = -by^1$ as a transformation on the XY^1 plane giving us a slant XY^1 plane. As shown in fig 2.3, the slanted Y axis is given by $y^1 = bx$ and slanted X axis is given by $x = -by^1$.

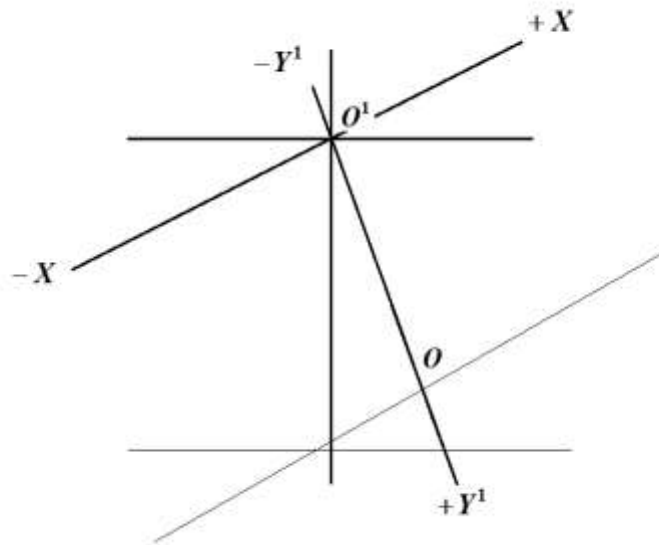


Fig2.3; the slanting of the XY^1 plane

STEP 5: Draw curve $y^1 = ax^2$ as derived from the component function $f_2(x) = ax^2$ on the XY^1 plane derived.

The curve $y^1 = ax^2$ gives some X value on the line $y^1 = \frac{b^2}{4a}$ for the slanted XY^1 plane which happens to be the

line $y = 0$ for the initially arbitrary chosen origin O , the X value are found to be $x = \frac{b}{2a}$ and $x = -\frac{b}{2a}$. Also from

condition 3, given the gradient on the normal of $y^1 = bx$ to be $x = -by^1$, in the case negative we choose the X value with negative gradient on the curve $y^1 = ax^2$ as the zero of the correspondent perfect square. Hence from

$$\frac{F^1(x)^2}{4a} = ax^2 + bx + \frac{b^2}{4a} \text{ the } X \text{ value is found to be } x = -\frac{b}{2a}.$$

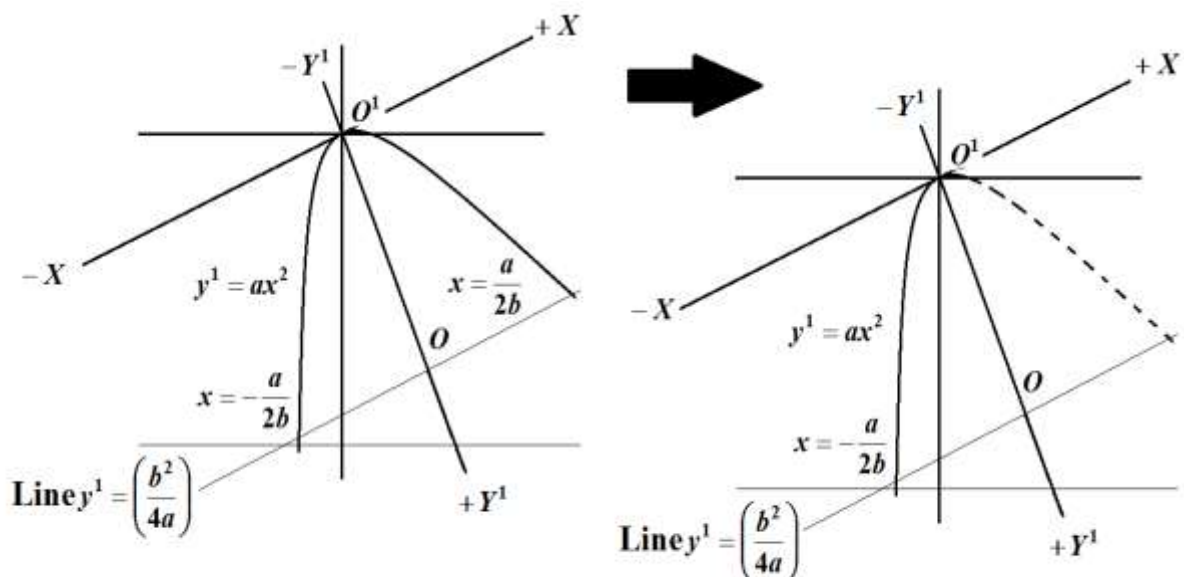


Fig 2.4; The sort for the zero of the correspondent perfect square.

STEP 6: Now back to the equation given as $F(x) = ax^2 + bx + c$, with this $F(x)$ is separated into its component functions. Thus, $f_0(x) = c$, $f_1(x) = bx$ and $f_2(x) = ax^2$.

Then from the formula $y^1 = \frac{b^2}{4a} - y$ draw line $y^1 = \frac{b^2}{4a} - c$ in reference to the point O^1 or in other words draw line $y = c$ in reference to point O on the curve $y^1 = ax^2$ already drawn from the correspondent perfect square in step 5.

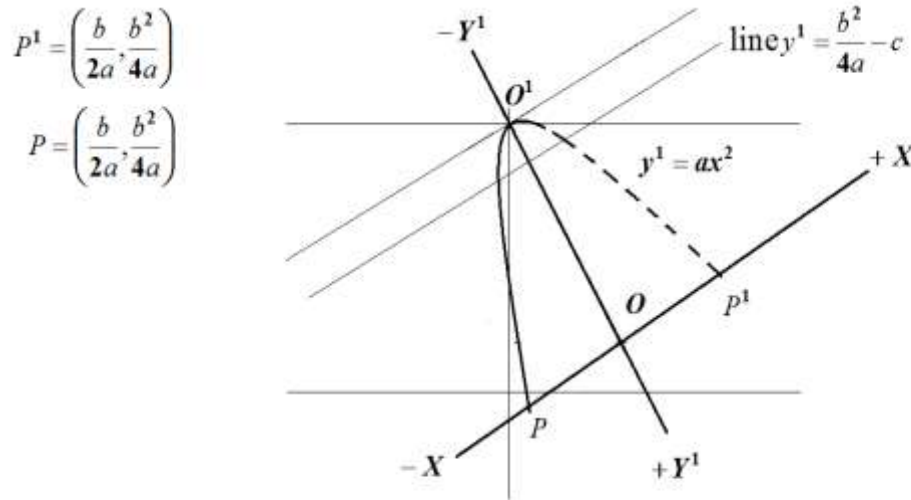


Fig 2.5; The representation of the line $y^1 = \frac{b^2}{4a} - c$ of $F(x)$ on the curve $y^1 = x^2$ of the correspondent perfect square.

STEP 7: Maintain the initial geometry of $f_1(x) = bx$ and $f_2(x) = ax^2$ by not redrawing them for $F(x)$ as was already done by the correspondent perfect square $\frac{F^1(x)^2}{4a}$. The component functions $f_1(x) = bx$ and $f_2(x) = ax^2$ are not drawn from the origin O in the XY plane, because in doing so, useful intersections in the line $y = 0$ would be guaranteed.

STEP 8: After the line $y^1 = \frac{b^2}{4a} - c$ is drawn as shown in STEP 6, some intersections obtained between the line

$y = c$ or $y^1 = \frac{b^2}{4a} - c$ and the curve $y^1 = ax^2$ results in new axes at that point of intersection to cause a translation of the curve $y^1 = ax^2$ in either side of the X axis. Thus, the curve is translated as was explain in **condition 2**. Again four intercepts on line $y = 0$ or line $y^1 = \frac{b^2}{4a}$ are represented as points M, M^1, N and N^1 .

Where M has $x = -\frac{b}{2a} - \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ and N has $x = -\frac{b}{2a} + \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ with negative gradient on the curve

and N^1 has $x = \frac{b}{2a} + \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ and M^1 has $x = \frac{b}{2a} - \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ with positive gradient on the curve $y^1 = ax^2$.

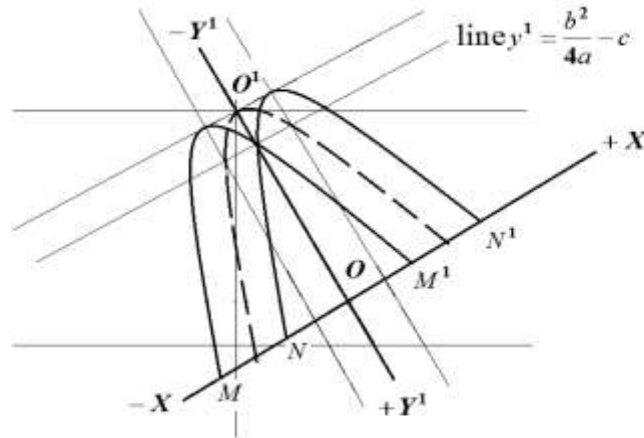


Fig2.6; the shifting of the curve $y^1 = ax^2$

STEP 9: The roots of the quadratic equation is found with reference made by the sign on the gradient of the four intercepts with the sign on the normal $x = -by^1$ which in this case is known to be negative, so negate the X values which are having positive gradient on the curve making them conform to the actual roots. Therefore

negating $x = \frac{b}{2a} + \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ and $x = \frac{b}{2a} - \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$. Surely the roots are $x = -\frac{b}{2a} - \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ and

$x = -\frac{b}{2a} + \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ in conformity to the X values with negative gradient on the curve $y^1 = ax^2$. Know that

if the normal had a positive gradient then the X values with positive gradient would be picked as the roots.

These values $x = -\frac{b}{2a} - \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ and $x = -\frac{b}{2a} + \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ collectively give the formula

$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b^2}{4a^2} - \frac{c}{a}\right)}$ which is known as the quadratic formula written popularly as $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

Hence the quadratic formula is proved.

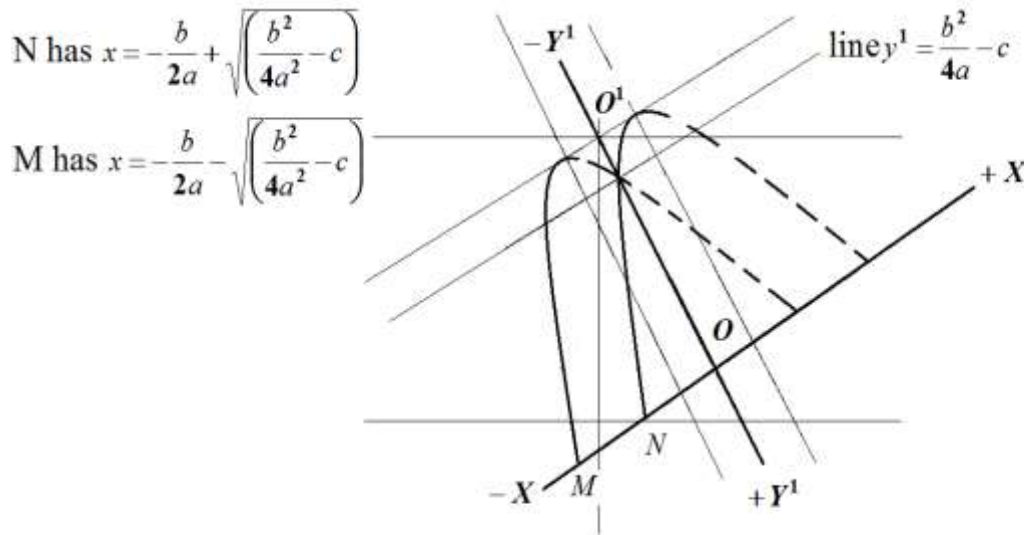


Fig2.7; the determination of the roots at point M and N

EXAMPLES

EXAMPLE 1: Finding the roots of a given quadratic equation $F(x) = x^2 + 2x - 3$.

STEP 1: the correspondent perfect square can be found as;

$$\frac{F^1(x)^2}{4} = x^2 + 2x + 1$$

Hence the component functions of this equation $0 = x^2 + 2x + 1 - y$ are operated with in the succeeding steps.

STEP 2: The component functions are defined as $f_0(x) - y = 1 - y$, $f_1(x) = 2x$ and $f_2(x) = x^2$.

STEP 3: Translate the XY plane to XY^1 plane with $y^1 = 1 - y$ and observe that the origin on the XY plane is $O = (0,0)$ and that on the XY^1 plane is $O^1 = (0,0)$.

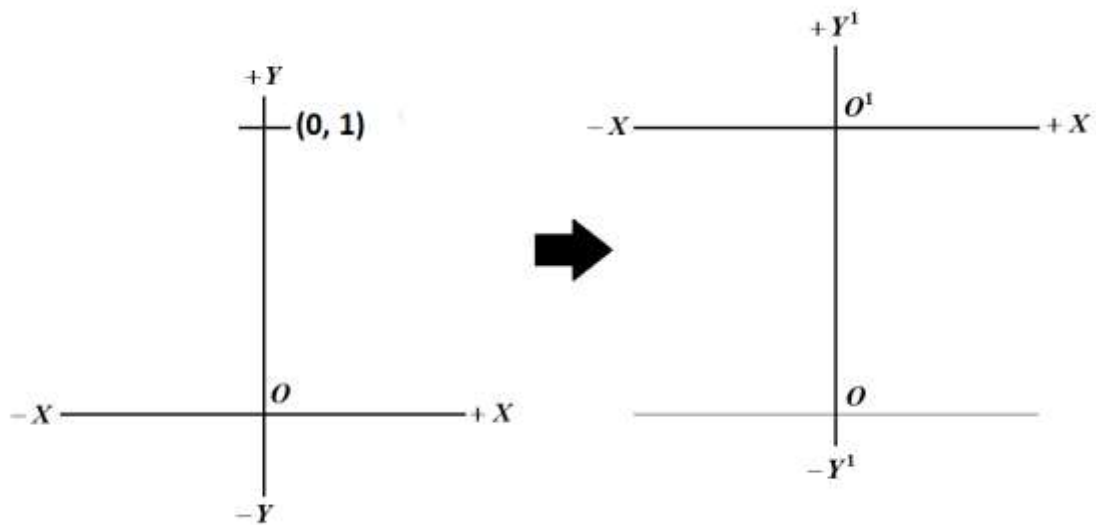


Fig 2.8; The resolution of the XY^1 plane from the arbitrary XY plane

STEP 4: Draw line $y^1 = 2x$ from the component function $f_1(x) = 2x$, and its perpendicular line $x = -2y$ to slant the XY^1 plane.

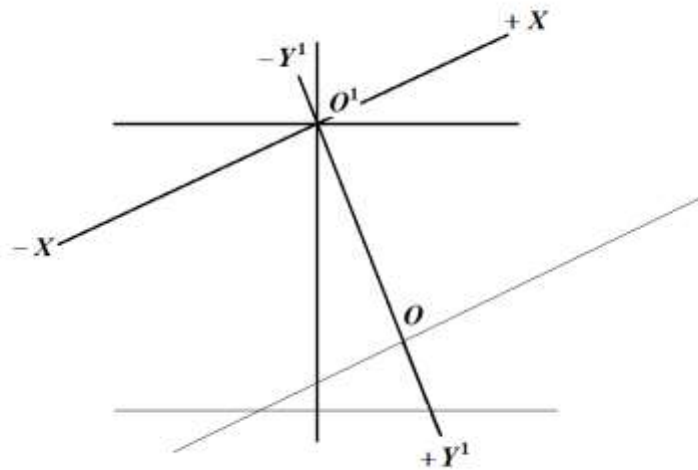


Fig 2.9; The slant XY^1 plane

STEP 5: Draw curve $y^1 = x^2$ as obtained from the component function $f_2(x) = x^2$ of the correspondent perfect square on the XY^1 plane derived. The curve $y^1 = x^2$ gives some intersections on the line $y^1 = 1$, which happen to be at line $y = 0$ for the initially arbitrary chosen origin O . Hence based on the intersections, the X intercept are found to be $x = 1$ and $x = -1$ also by knowing that from step 4, line $x = -2y$ has a negative gradient so therefore the X intercept with negative gradient on the curve $y^1 = x^2$ is chosen, as shown below in fig 2.10 likewise from **condition 3**. Hence the roots of the correspondent perfect square $\frac{F^1(x)^2}{4} = x^2 + 2x + 1$ is found to be $x = -1$.

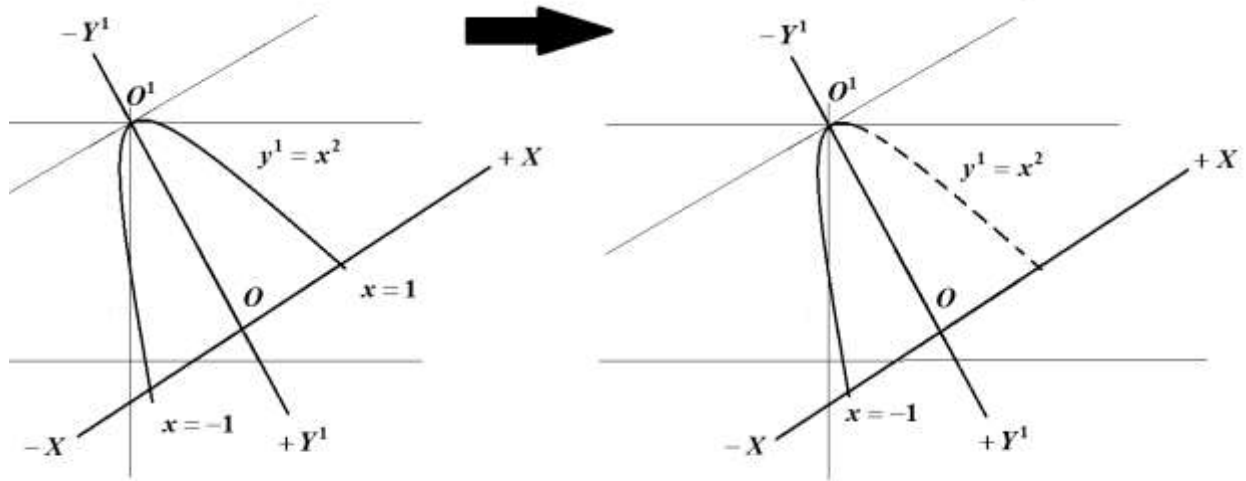


Fig 2.10; The representation of $y^1 = x^2$ on the XY^1 plane and the sort for the roots of the correspondent perfect square

STEP 6: Now back to the equation given as $F(x) = x^2 + 2x - 3$, the component functions of this given equation is defined as $f_0(x) = -3$, $f_1(x) = 2x$ and $f_2(x) = x^2$. Taking $y = -3$ from component function $f_0(x) = -3$ in reference to point O , $y = -3$ is substituted into $y^1 = 1 - y$, such that line $y^1 = 4$ is obtained in reference to the point O^1 and this line is drawn on the curve $y^1 = x^2$ already drawn from the correspondent perfect square.

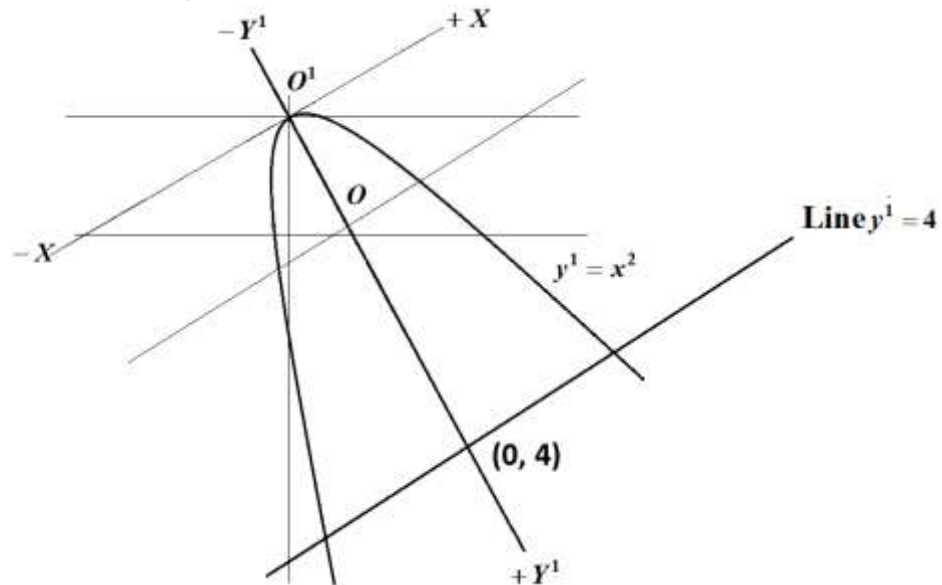


Fig 2.11; The representation of the line $y^1 = 4$ on the curve $y^1 = x^2$ of the correspondent perfect square.

STEP 7: Do not draw $f_1(x) = 2x$ and $f_2(x) = x^2$ again for the quadratic equation $F(x) = x^2 + 2x - 3$.

STEP 8: After the line $y^1 = 4$ is drawn as shown in STEP 6, some intersections obtained between the line $y^1 = 4$ and the curve $y^1 = x^2$ results in new axes at that point of intersection to cause a translation of the curve $y^1 = x^2$ into the curves $y^1 = (x - 2)^2$ and $y^1 = (x + 2)^2$. Again four intercepts on line $y = 0$ or $y^1 = 1$ are

represented as point M, M^1, N and N^1 . Where M has $x = -3$ and N has $x = 1$ with negative gradient on the curve, also N^1 has $x = 3$ and M^1 has $x = -1$ with positive gradient on the curve $f_2(x) = x^2$.

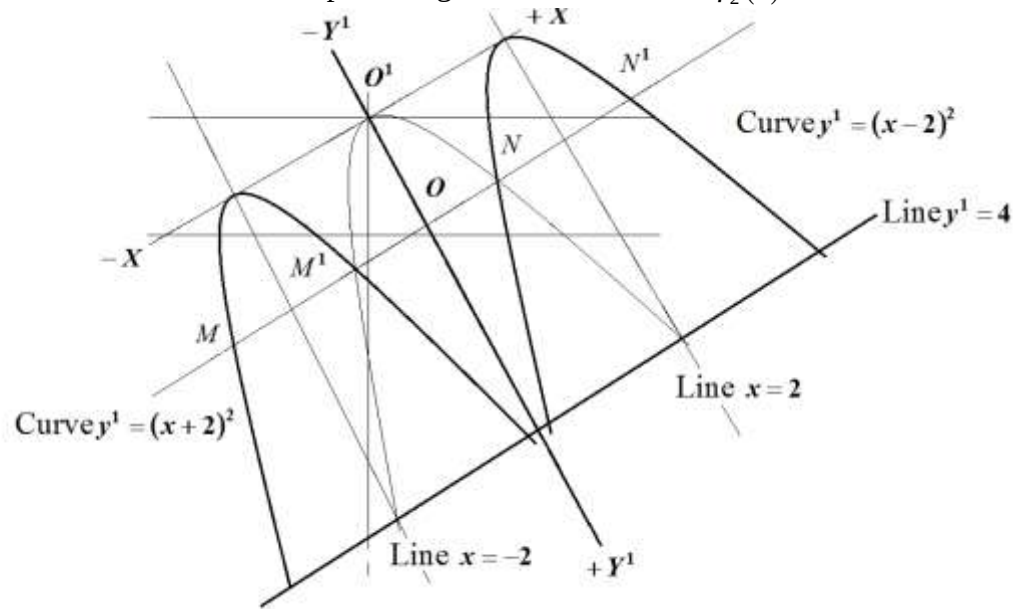


Fig 2.12; The Translation of the curve $y^1 = x^2$

STEP 9: The roots of the quadratic equation is found with reference made by the sign on intercepts with the sign on the normal which in this case is known to be negative, so negate the X values which are having positive gradient on the curve making them conform to the actual roots. Therefore negating $x = 3$ and $x = -1$, the roots are found to be $x = -3$ and $x = 1$ in conformity to the X values with negative gradient on the curve $y^1 = x^2$.

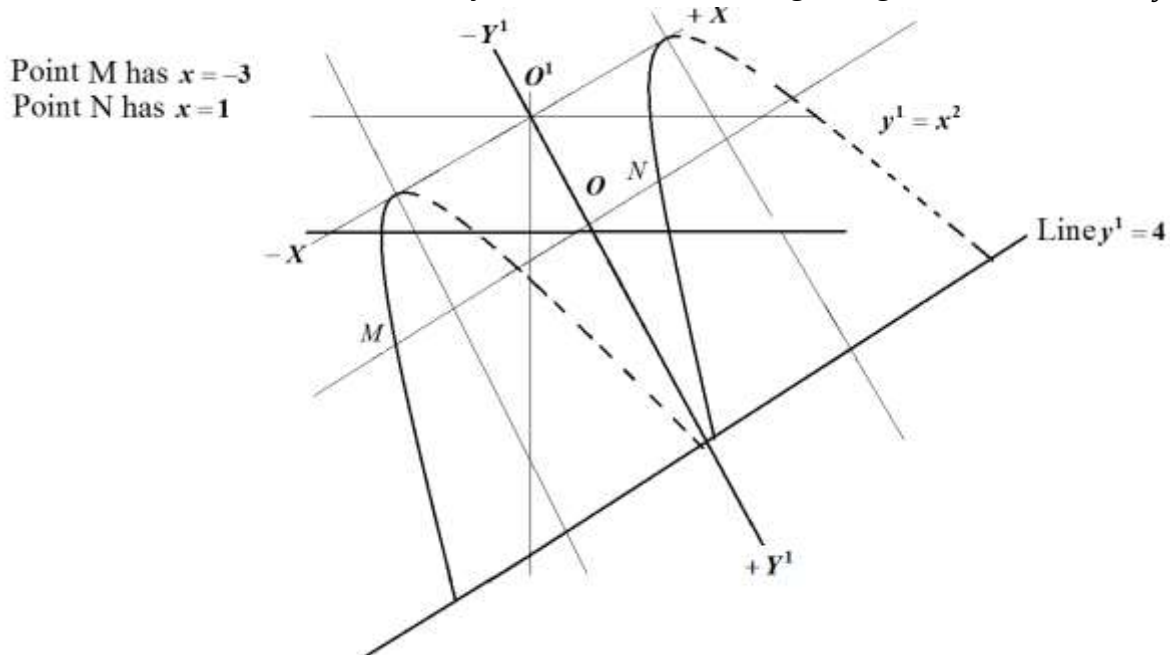


Fig 2.13; The sort for the roots of the equation in question

EXAMPLE 2: Finding the root of a given quadratic function $F(x) = x^2 - 4$.

STEP 1: the derivation of the correspondent perfect square.

Which is $\frac{F^1(x)^2}{4} = x^2$.

Making $\frac{F^1(x)^2}{4} = y$ then $0 = x^2 - y$ is obtained which is operated on in the succeeding steps.

STEP 2: the component functions of the correspondent perfect square are defined as $f_0(x) = -y$, $f_1(x) = 0$ and $f_2(x) = x^2$.

STEP 3: Translate the XY plane to XY^1 plane with $y^1 = -y$. Here point O^1 coincides with point O in other words; the position of point O is occupied by point O^1 . Hence the arbitrary origin $O(0, 0)$ is the same as $O^1(0, 0)$ on the XY^1 plane.

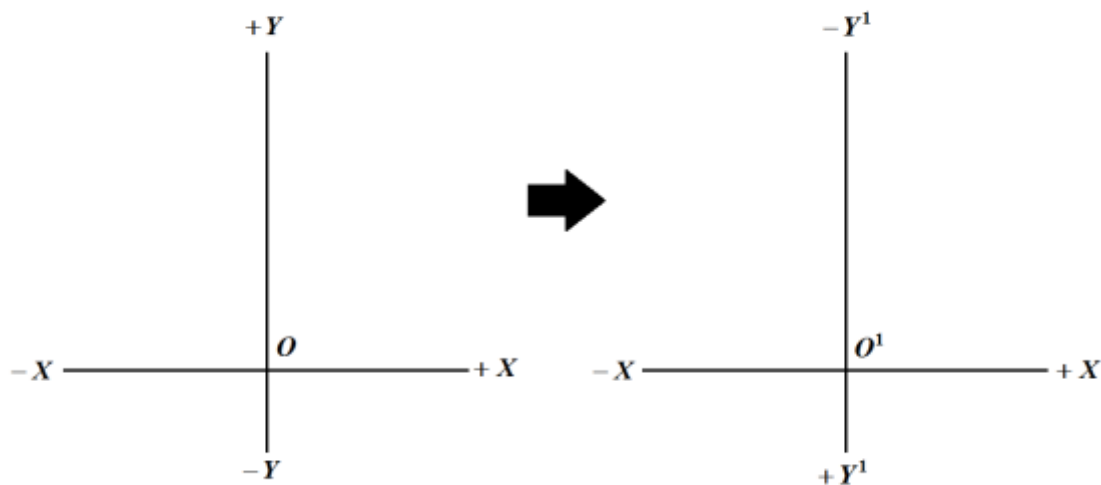


Fig 2.14; The resolution of the XY^1 plane from the arbitrary XY plane

STEP 4: Maintain the geometry of the XY^1 plane, as the component function $f_1(x) = 0$ has $y^1 = 0$ therefore there is no slanting of the XY^1 plane.

STEP 5: Draw $y^1 = x^2$ on XY^1 plane. Note that the intersection of the curve with the X axis would locate the zeroes of the correspondent perfect square hence the zero is $x = 0$.

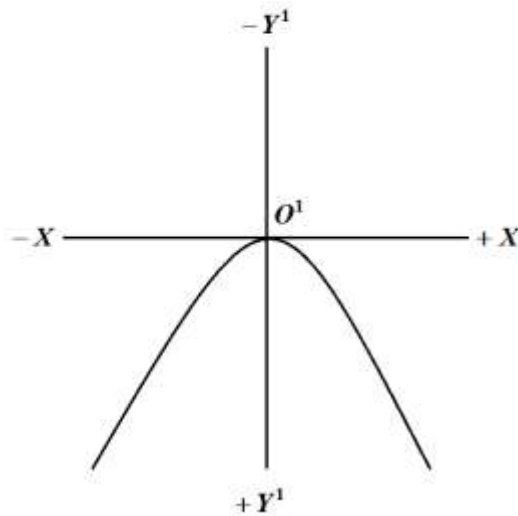


Fig 2.15; The representation of $y^1 = x^2$ on XY^1 plane

STEP 6: After a successful geometric representation of the correspondent perfect square focus on the originally given function $F(x) = x^2 - 4$ then draw in the XY^1 plane line $y^1 = 4$ given the formula $y^1 = -y$ obtained from the correspondent perfect squares as $y = -4$ from $F(x)$.

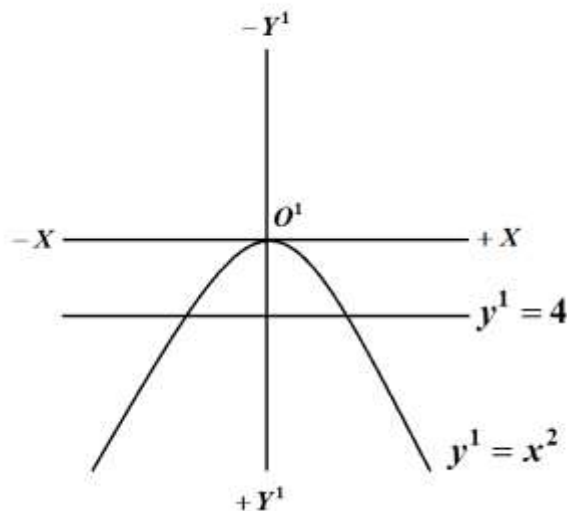


Fig 2.16; The representation of the line $y^1 = 4$ on the curve $y^1 = x^2$ of the correspondent perfect square.

STEP 7: Maintain the initial geometry of $f_1(x) = 0$ and $f_2(x) = x^2$ of the given function $F(x)$ which are the same component functions as that of the correspondent perfect square.

STEP 8: After line $y^1 = 4$ is drawn in step 6, some intersections obtained between the line $y^1 = 4$ and the curve $y^1 = x^2$ result in new axes at those points of intersection to cause a translation of the curve $y^1 = x^2$ into the curves $y^1 = (x - 2)^2$ and $y^1 = (x + 2)^2$. The X intercepts $x = 2$ and $x = -2$ are the roots.

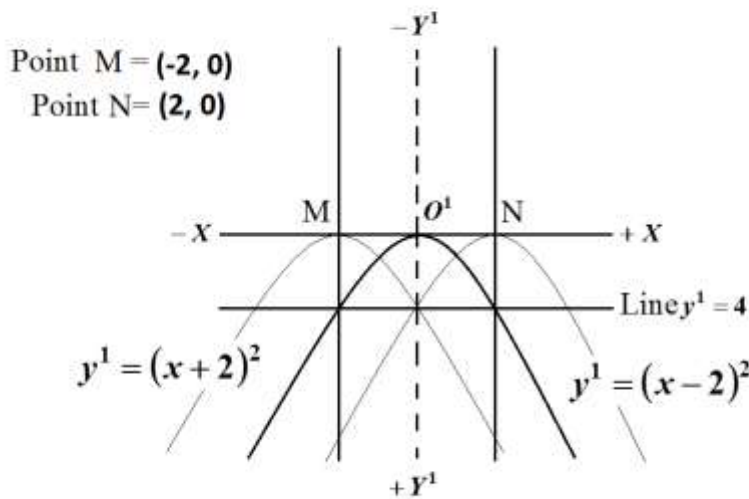


Fig 2.17; The sort for the roots of the function $F(x)$.

STEP 9: Then with the slope of the normal which happens to be $x = 0$ that is neither negative nor positive the roots of the quadratic equation $F(x) = x^2 - 4$ are therefore maintained as $x = 2$ and $x = -2$. Take reference from the origin O which coincides with O^1 when defining your roots.

3. RESULTS.

- This method renders a proof for the quadratic formula. so, being able to prove the quadratic formula, this method would be able to find the roots of all quadratic equations.
- The slanting by $y^1 = a_1x^1$ and its perpendicular $x = -a_1y^1$ offers the ability to analyze the XY plane together with the XY^1 plane simultaneously as the resultant diagram is equivalent to Euclid's fifth postulate.
- This method offers a purely geometric method in roots determination.

4. DISCUSSION.

Not only does the discussion focuses on what the result given above mean, but elaborates the points in the results and throws more light on the whole content of this text. This study has been able to derive a proof for the quadratic formula using the function form $F(x) = ax^2 + bx + c$ as popularly known to be the general formula for quadratic equations. The quadratic formula was proved following **section 2.1**.

In addition, due to the facts that this method is able to prove the quadratic formula therefore it is liable to solve all quadratic equations. As a privilege this method works effectively with the terms of a particular quadratic equation to locate its respective roots and then now there is an advancement in the modern day perception of the way we see quadratic curve so there would be great enlightenment in other higher polynomial equations.

4.1 The Illustration of Euclid's Fifth Postulate.

From step 4 in the procedure and the proof of the quadratic formula, the diagram is observed to obey the parallel postulate by comparison with the actual diagram of the parallel postulate in **fig1.1**. This says that, if the sum of the interior angles α and β is less than 180° then the two straight lines, produced indefinitely, meet at some point.

From the slant XY^1 plane below in **fig 3.1**, angle α (measured in degrees) is known to be the angle between line $y^1 = \frac{(a_1)^2}{4a_2}$ and line $y^1 = a_1x^1$ and angle β (measured in degrees) is known to be the angle between line $y^1 = \frac{(a_1)^2}{4a_2}$ and line $x = -a_1y$. The line $y^1 = a_1x^1$ and line $x = -a_1y$ meet at O^1 .

So the two lines drawn, line $y^1 = a_1x^1$ and line $x = -a_1y$, which intersect a third, line $y^1 = \frac{(a_1)^2}{4a_2}$, in such a way that the sum of the inner angles $\alpha + \beta$ on one side is less than two Right Angles, 180° , then the two lines inevitably intersects each other on that side as extended far enough. The diagram in step 4 of the procedure and the proof of the quadratic formula is equivalent to what is known as the Parallel Postulate. In a nutshell, there is an observation that the existence of the lines as explained in the parallel postulate come by the translation undergone by the original XY plane and the XY^1 plane, which result in an intersection of the X axis and X^1 axis and also the Y^1 axis and the X axis, but relating to the explanation in this section we consider solely the intersection of the Y^1 axis and the X axis due to conveniences and their intersection at the origin O^1 . The third axis being the original X axis guarantee a sum of interior angle (α and β) less than two 90° between each of the two axes and the original X axis.

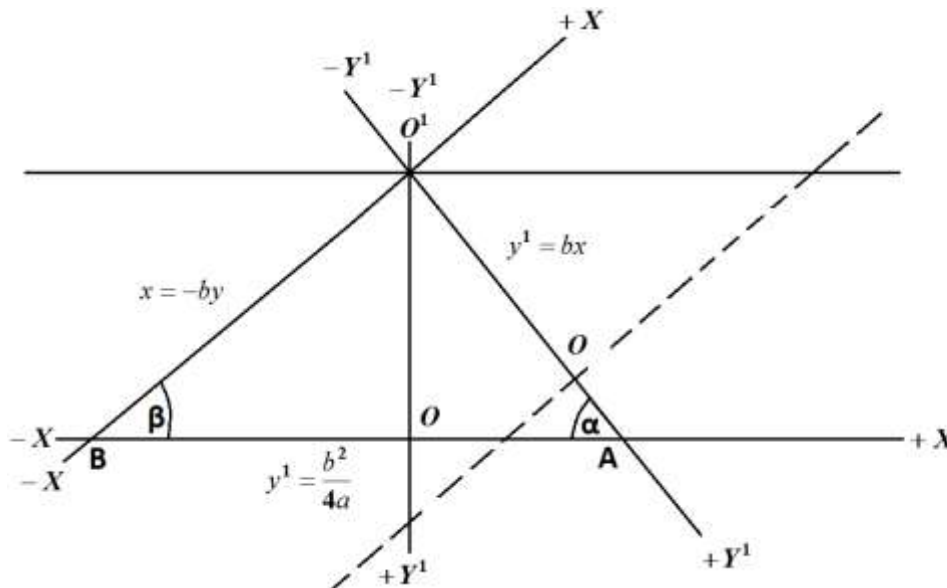


Fig 3.1 the diagram indicating the parallel postulate

5. CONCLUSION

- From step 4 to step 10 of the procedure and the proof of the quadratic formula, it is right to say that the points on a quadratic curve have some frame work which clearly depict Euclid's fifth postulate.
- As this method is able to prove the quadratic formula, it is right to say that the study renders an alternative roots determination method.
- In addition to the understanding of curves (quadratic curves) there is a theory that, by basically taking the terms of some function separately the roots of that function can be determined.
- Also from the point of view of this method, there are more to the graphs usually drawn which are somewhat straight forward in nature that there is no revelation of the (discriminant and average) parts of the quadratic formula, which this new finding does. In other words, the graphs normally drawn are like God's inner complicated portions of a complex machine which is perceived to be its model.

6. RECOMMENDATIONS

The main idea from this article is recommended for other higher researches in all area of polynomials equations, as it is basically Euclid characteristic. Again its shows correctness as far as the proof of the quadratic formula is concerned, and interestingly it can be done without any numeric calculations giving it an upper hand over other iterative method whose approximate values of the roots are not properly predicted.

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